Key Difference Invariant Bias in Block Ciphers

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Linear cryptanalysis

Linear cryptanalysis was introduced by Matsui in 1992.

Based on the notion of linear approximations:

Linear approximation (a, b) of a vectorial function F:

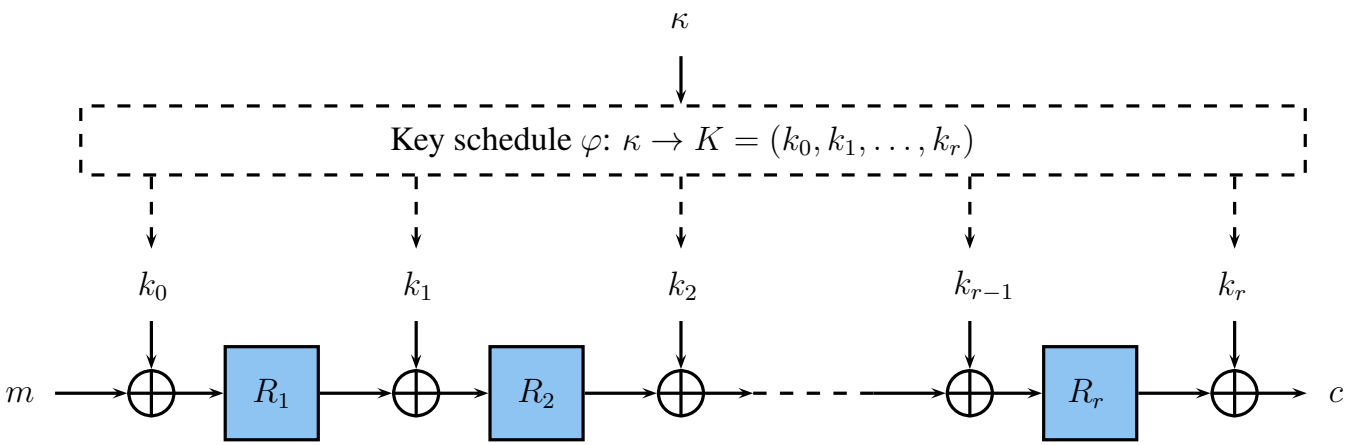
$$b \cdot F(x) \oplus a \cdot x.$$

Bias of (a, b):

$$\varepsilon_{a,b}^{F} = Pr_x\{b \cdot F(x) \oplus a \cdot x\} - \frac{1}{2}$$

Key-alternating block ciphers

Many block ciphers belong to this class: AES, PRESENT,...



- κ: user-supplied key
- K: expanded key
- $k_0, \ldots, k_r$: round subkeys
- n: block size

Linear characteristics and linear hulls

A linear approximation (a, b) of an iterated block cipher is called a

linear hull.

Composed of all the linear approximations θ over individual rounds, with input mask a and output mask b:

$$\theta = (\theta_0, \theta_1, \ldots, \theta_{r-1}, \theta_r),$$

with $\theta_0 = a$ and $\theta_r = b$.
Bias ε_θ of the linear characteristic θ:

$$\varepsilon_\theta = 2^{r-1} \prod_{i=1}^{r} \varepsilon_{\theta_{i-1},\theta_i}.$$

Case of key-alternating ciphers

Only the sign of ε_θ depends on the key value.

Proposition [DR02]: For a key-alternating block cipher, the bias ε of a linear hull (a, b) is

$$\varepsilon = \sum_{\theta:\theta_0=a,\theta_r=b} (-1)^{d_\theta+\theta\cdot K}|\varepsilon_\theta|,$$

where d_θ : $\varepsilon_\theta[0] = (-1)^{d_\theta}|\varepsilon_\theta|$.

Fundamental Question

How does the bias of a linear hull behave under a change of key?

Our contribution:

The bias of a linear hull can be invariant
under a change of the key.

Towards bias invariance under key difference

- Let a linear hull (a, b)
- Let ε and ε' two biases under two keys κ and κ', with $\kappa \neq \kappa'$.

When $\varepsilon = \varepsilon'$?

- Let $K = \varphi(\kappa)$, $K' = \varphi(\kappa')$ and $K \oplus K' = \Delta$.

Then,

$$\varepsilon = \sum_{\theta:\theta_0=a,\theta_r=b} (-1)^{d_\theta+\theta\cdot K}|\varepsilon_\theta| \text{ and } \varepsilon' = \sum_{\theta:\theta_0=a,\theta_r=b} (-1)^{d_\theta+\theta\cdot K'}|\varepsilon_\theta|$$

Equality holds if

$$d_\theta + \theta \cdot K = d_\theta + \theta \cdot K'.$$

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$$\theta \cdot (K \oplus K') = 0.$$

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Equality holds if

$$\theta \cdot \Delta = 0.$$

Key-difference invariant bias for key-alternating ciphers

Theorem

Let (a, b) with $a, b \neq 0$ be a linear hull of a key-alternating block cipher. Let ε and ε' be the biases for the expanded keys K and K' respectively, with $K \oplus K' = \Delta$. Then

$$\varepsilon = \varepsilon' \quad \textit{if} \quad \theta \cdot \Delta = 0$$

for each *linear characteristic θ of the linear hull (a, b) with $\varepsilon_\theta \neq 0$.*

How to construct the property in practice?

Call zero position a bit position $\theta(j), j = 1, \ldots, n(r+1)$ such that

$$\theta(j) = 0 \text{ for all linear approximations } \theta \text{ with } \varepsilon_\theta \neq 0.$$

Choose $\Delta = K \oplus K'$ in a way that $\Delta(j)$ is

- active in some zero-positions and
- non-active in all nonzero-positions.

The example of AES-256 (1)

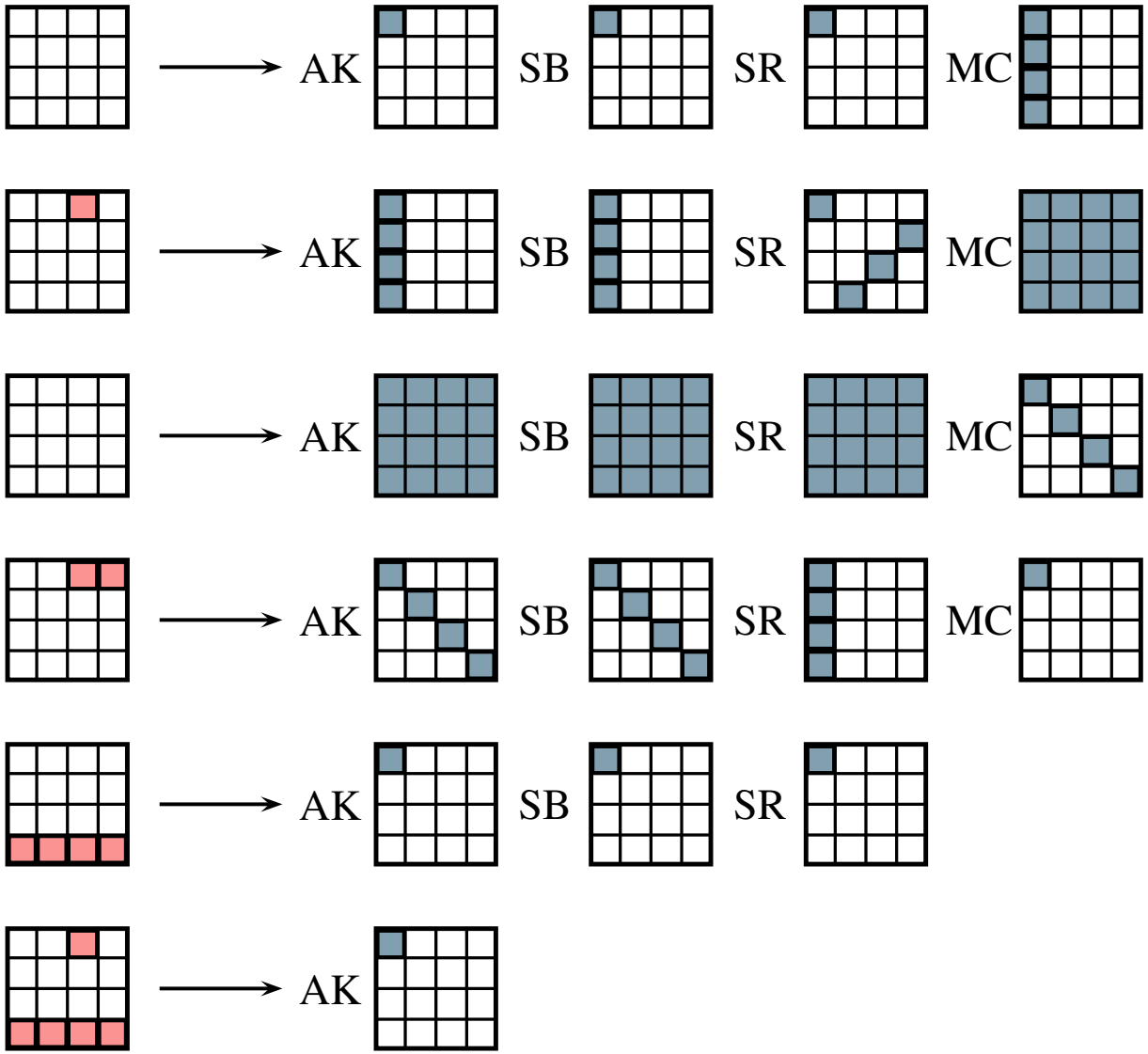
Consider two 32-byte master keys κ and κ' with

$$\kappa \oplus \kappa' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \boldsymbol{\delta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Define the input mask a and the output mask b as:

$$a = \begin{bmatrix} \boldsymbol{\alpha} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} \boldsymbol{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0. \end{bmatrix}$$

The example of AES-256 (2)



Key invariance bias and idealized cipher

For a randomly selected cipher:

$$\Pr\{\varepsilon = \varepsilon' | \kappa \neq \kappa'\} \approx \frac{1}{\sqrt{2\pi}} 2^{\frac{3-n}{2}}, \text{ for } n \geq 5.$$

Distinguisher

Underlying statistic:

$$s = \sum_{i=1}^{\ell} \left[\left(\frac{S_i}{N} - \frac{1}{2} \right) - \left(\frac{S_i'}{N} - \frac{1}{2} \right) \right]^2.$$

- N: # of plaintexts/ciphertexts
- ℓ: # linear approximations under (K, K') with $K \oplus K' = \Delta$.
- S_i, S_i': counters.

Two distinct distributions:

- $s \sim \mathcal{N}\left(\frac{\ell}{2N}, \frac{\ell}{2N^2} \right)$ for the right key.
- $s \sim \mathcal{N}\left(\frac{\ell}{2N} + \frac{\ell}{2^{n+1}}, \frac{\ell}{2N^2} + \frac{\ell}{2^{2n+1}} + \frac{\ell}{N2^n} \right)$ for the wrong key.

Data complexity of the distinguisher

Condition for the distinguisher to work:

$$q_{1-\alpha_1}\left(\frac{\ell}{2N^2}+\frac{\ell}{2^{2n+1}}+\frac{\ell}{N2^n}\right)+q_{1-\alpha_0}\frac{\ell}{2N^2}=\frac{\ell}{2^{n+1}}.$$

- $q_{1-\alpha_1}$ and $q_{1-\alpha_0}$: quartiles of standard normal distribution.
- α_0: failure probability of the attack.
- α_1: proportion of surviving keys.

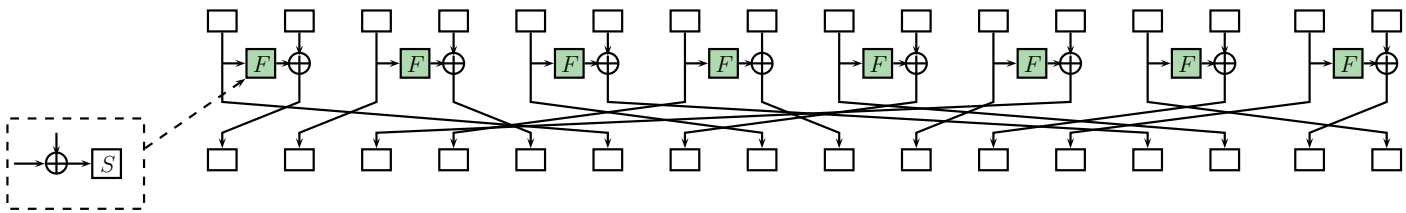
Data complexity:

$$N=\frac{2^{n+0.5}}{\sqrt{\ell}}-q_{1-\alpha_1}\sqrt{2}(q_{1-\alpha_0}+q_{1-\alpha_1}).$$

TWINE-128

Proposed by Suzaki, Minematsu, Morioka and Kobayashi [SAC '12].

- 64-bit blocks, 128-bit keys.
- 36 rounds.



Key schedule: Non-linear, nibble-oriented.

Best previous cryptanalysis: 24-round impossible differential attack in the single key model [SMMK '12].

Linear approximations with key difference invariant bias

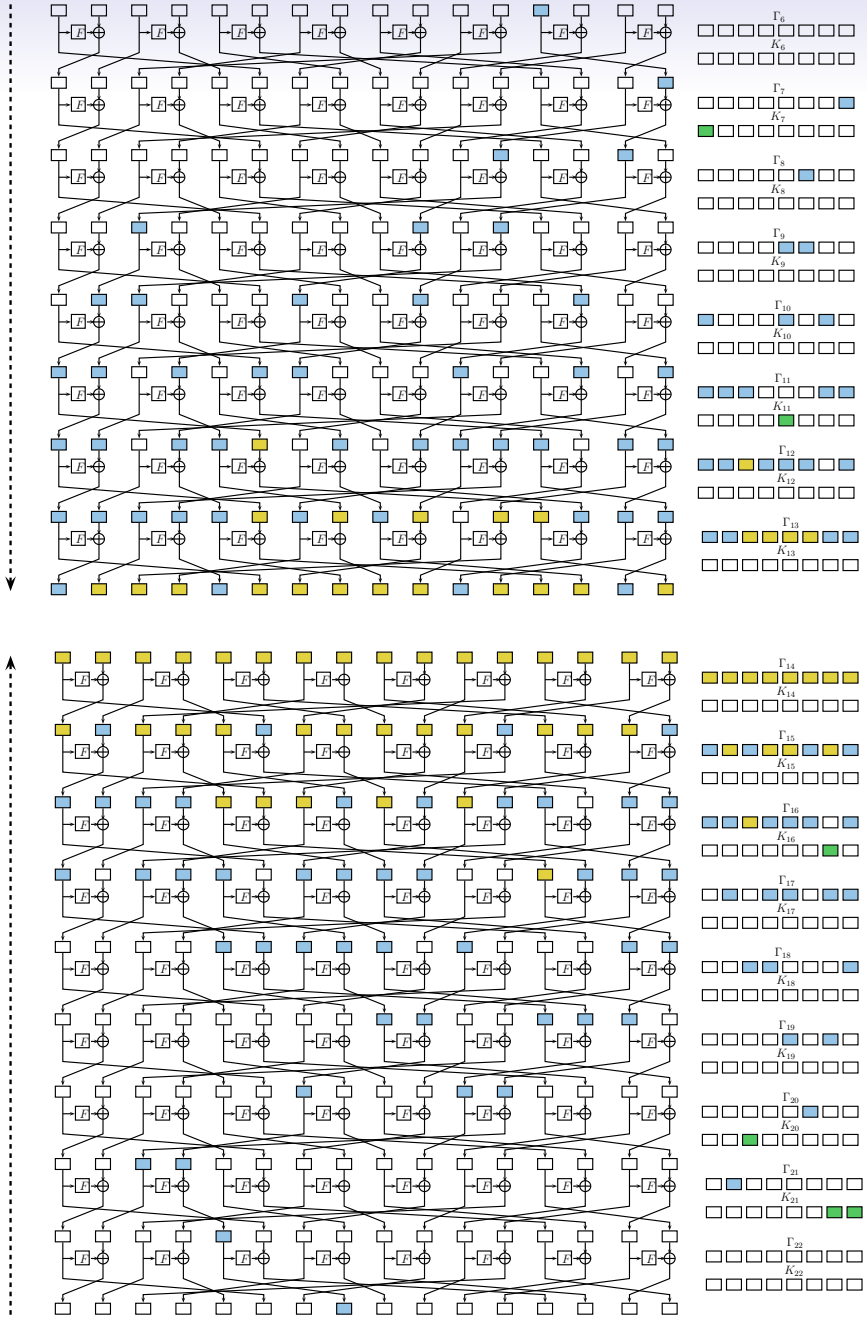
- 17-round linear approximations with key invariant bias under related-key differential paths.
- Input/output masks :

$$(000000000000\alpha 000, 0000000\beta 00000000),$$

with $\alpha, \beta \neq 0$.

$\Rightarrow 15 * 15 \approx 2^{7.81}$ linear approximations.

- Key difference on bits $104 - 107$ of the master key.


Γ_6
K_6
Γ_7
K_7
Γ_8
K_8
Γ_9
K_9
Γ_{10}
K_{10}
Γ_{11}
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K_{20}
Γ_{21}
K_{21}
Γ_{22}
K_{22}

Key-recovery for 27-round TWINE

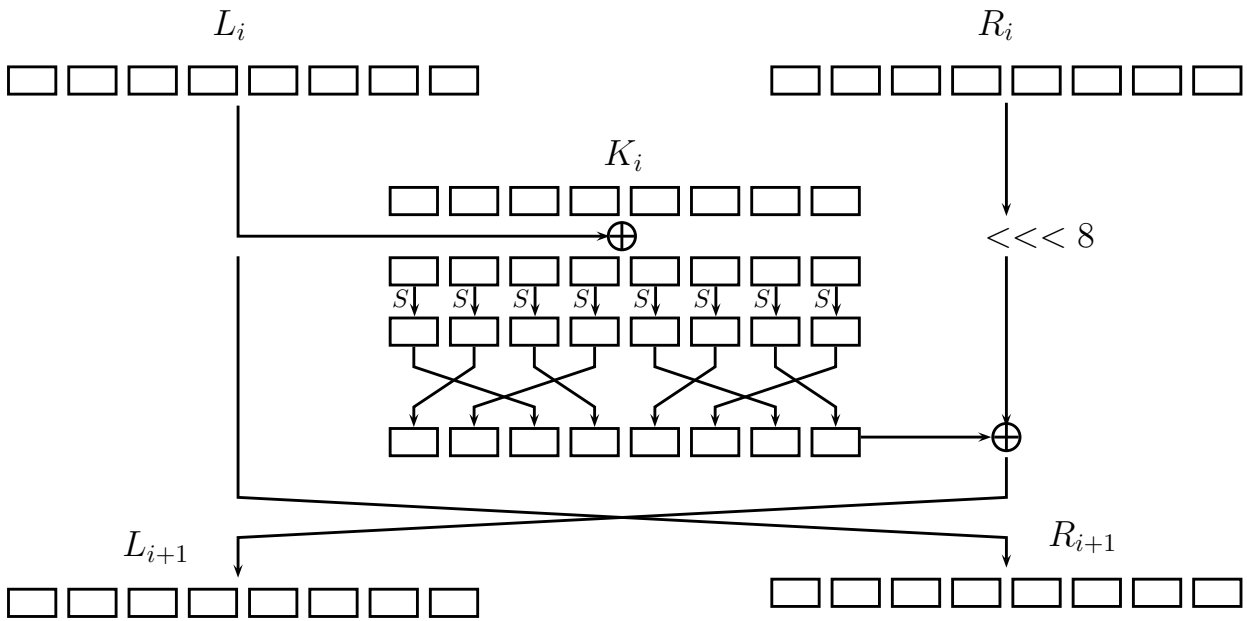
- Append 5 rounds at the beginning and 5 rounds at the end of the characteristic.

α_0	α_1	Data (KP/key)	Time (27R enc.)	Memory (Bytes)
$2^{-2.7}$	$2^{-8.5}$	$2^{62.95}$	$2^{119.5}$	2^{61}
$2^{-2.7}$	$2^{-4.5}$	$2^{62.29}$	$2^{123.5}$	2^{61}

LBlock

Designed by Wu and Zhang, (ACNS 2011).

- 80-bit key and 64-bit state.
- 32 rounds.



LBlock-Summary of Attacks

Model	Attack	#Rounds	Data per key	Time	Memory	Ref.
Single-key	Imp. Diff	20	2^{63} CP	$2^{72.7}$	2^{68}	[WZ11]
	Imp. Diff	21	$2^{62.5}$ CP	$2^{73.7}$	$2^{55.5}$	[LGLL12]
	Imp. Diff	22	2^{58} CP	$2^{79.28}$	2^{76}	[KDH12]
	Integral	20	$2^{63.7}$ CP	$2^{63.7}$	N/A	[WZ11]
	Integral	20	$2^{63.6}$ CP	$2^{39.6}$	2^{35}	[SW12a]
	Integral	22	2^{61} CP	2^{70}	2^{63}	[SW12b]
	Zero-Correlation	22	$2^{62.1}$ KP	$2^{71.27}$	2^{64}	[SN12]
Related-key	Imp. Diff	22	2^{47} RKCP	2^{70}	N/A	[MN12]
	Differential	22	$2^{63.1}$ RKCP	2^{67}	N/A	[LGW12]
	Key Invariant Bias	**24**	$2^{62.29}$ **RKKP**	$2^{74.59}$	2^{61}	**This paper**
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- 16-round linear approximations with key invariant bias.

Conclusion

- New fundamental property of key invariant bias in key-alternating ciphers.
- Propose a statistical distinguisher for the property.
- News attacks in the related-key setting against LBlock and TWINE-128 covering more rounds than all previous attacks.

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Thanks for you attention!