Introduction to symmetric cryptography

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École de printemps en codage et cryptographie
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Overview

- **Introduction to symmetric-key cryptography**
- **Block ciphers**
- **Boolean functions and cryptographic Sboxes**
- **Attacks against block ciphers exploiting a low algebraic degree**
  - Algebraic attacks
  - Higher-order differential attacks
  - Integral attacks
- **Estimating the algebraic degree of iterated constructions**

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  - Higher-order differential attacks
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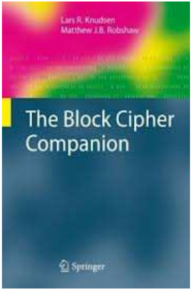
- **Estimating the algebraic degree of iterated constructions**

Bibliography

- *The Block Cipher Companion*, Lars Knudsen and Matt Robshaw

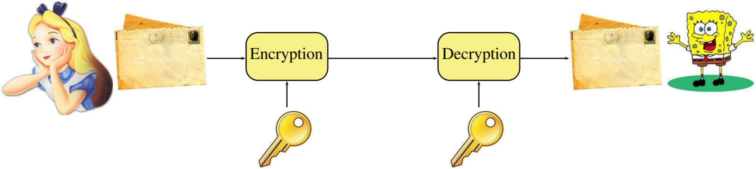


- *Lecture Notes on Cryptographic Boolean Functions*, Anne Canteaut
- *Analyse de Fonctions de Hachage Cryptographiques*, Thèse, Christina Boura

Outline

Symmetric-key encryption

Alice and Bob exchange the secret key through a **secure channel**.



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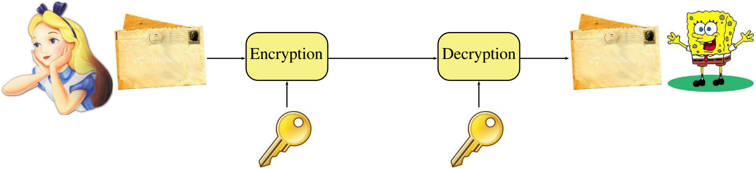
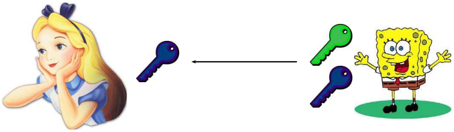
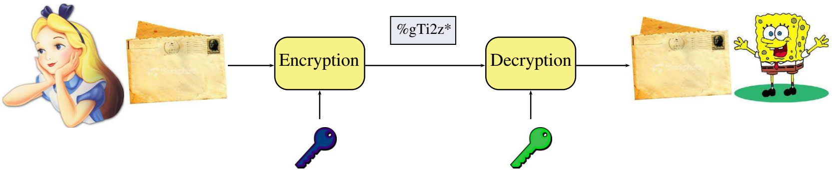


Key-exchange problem $\Rightarrow$ birth of the public-key cryptography.

Public-key encryption



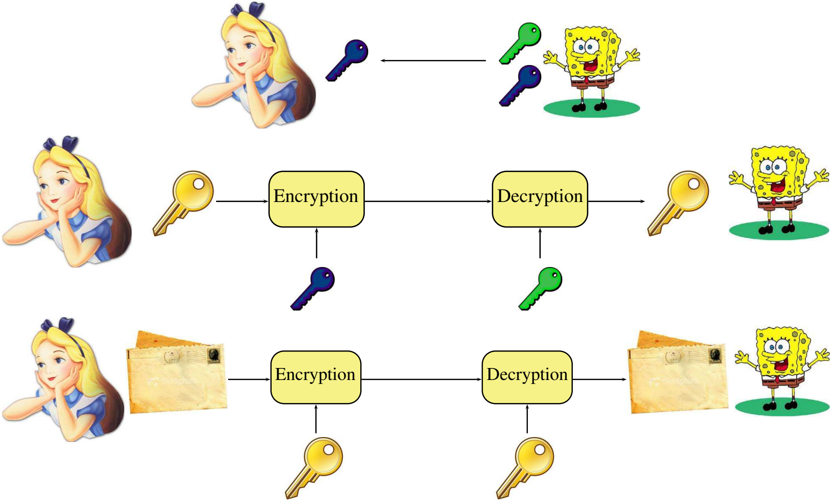


Advantages and disadvantages of each system

	Advantages	**Disadvantages**
Secret-key	Fast systems Relatively short-keys	Need secure key-exchange n users: $\frac{n(n-1)}{2}$ keys
Public-key	No key-exchange needed n users: $2n$ keys	Slow systems Relatively long-keys

Hybrid encryption

Idea: Use a combination of asymmetric and symmetric encryption to benefit from the strengths of every system.



Hybrid encryption

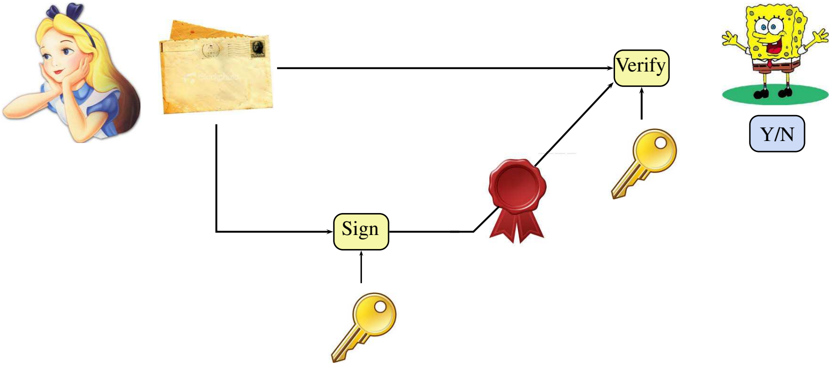
- Use a public-key cryptosystem to exchange a key (session key).
- Use the exchanged key to encrypt data by using a symmetric-key cryptosystem.

Advantages:

- Slow public-key cryptosystem is used to encrypt a short string only.
- Fast symmetric-key cryptosystem is used to encrypt the longer communication session.

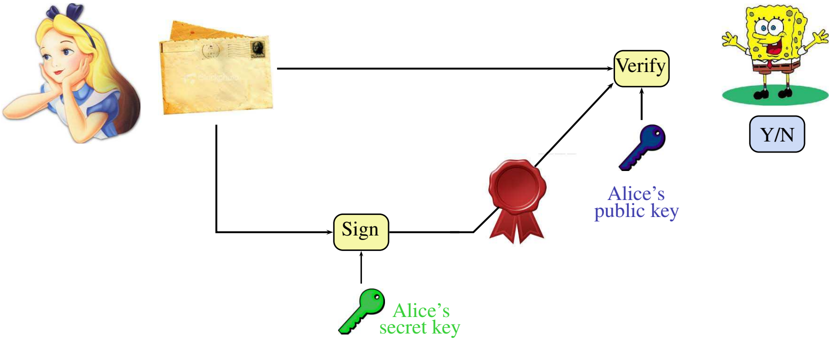
Symmetric-key authentication

Message authentication code (MAC)



Public-key authentication

Digital signatures



Hash functions

If the message to sign is long, the signing process becomes heavy...

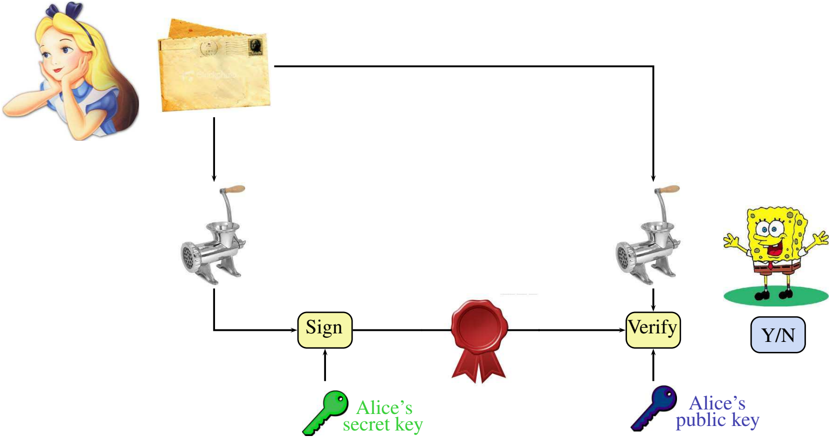
Idea: Use a cryptographic hash function.

$$H : \{0,1\}^* \to \{0,1\}^n$$

- A good hash function should be preimage, second-preimage and collision resistant.
- In recent hash proposals: $n = 256, 512$

Hash functions are considered as symmetric-key functions because they use similar building blocks with block-ciphers.

Hash and sign



The best of the two worlds

- Secrecy: Hybrid encryption
- Authentication: Digital signatures with hashing

There is a **need for both** public and symmetric-key cryptosystems.

Symmetric-key cryptosystems

A **cryptosystem** is a five-tuple $(\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D})$

- $\mathcal{P}$: set of possible plaintexts
- $\mathcal{C}$: set of possible ciphertexts
- $\mathcal{K}$: set of possible keys
- For each $k \in \mathcal{K}$, there is an encryption rule $e_k \in \mathcal{E}$ and a decryption rule $d_k \in \mathcal{D}$.

For each $k \in \mathcal{K} : d_k(e_k(m)) = m,$ for every $m \in \mathcal{P}$.

$$m \longrightarrow \boxed{e_k} \longrightarrow c \longrightarrow \boxed{d_k} \longrightarrow m$$

Kerckhoffs's principle (1883)

In 1883 August Kerckhoffs stated 6 design principles for military ciphers. The 2nd principle states:

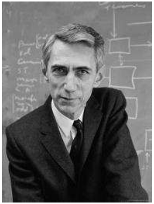
> *A cryptosystem should be secure even if everything about the system, except the key, is public knowledge.*

Reformulated by Claude Shannon as

"The enemy knows the system."

i.e., "*One ought design systems under the assumption that the enemy will immediately gain full familiarity with them.*"

Claude Shannon's theory



"Communication Theory of Secrecy Systems", published in 1949.

Many fundamental ideas of modern cryptography are introduced there:

- Provable security.
- Confusion and diffusion.
- Product ciphers.

Shannon's idea of perfect secrecy

"No information about the plaintext can be obtained by observing the ciphertext".

Shannon's definition:

A cryptosystem has perfect secrecy if

$$Pr(m|c) = Pr(m) \text{ for all } m \in \mathcal{P}, c \in \mathcal{C}.$$

An equivalent formulation:

$$Pr(c|m) = Pr(c) \text{ for all } m \in \mathcal{P}, c \in \mathcal{C}.$$

Shannon's theorem

A cryptosystem where $|\mathcal{P}| = |\mathcal{C}| = |\mathcal{K}|$ provides perfect secrecy iff

1. $Pr_{\mathcal{K}}(k) = 1/|\mathcal{K}|$, $\forall k \in \mathcal{K}$
2. $\forall m \in \mathcal{P}, c \in \mathcal{C}$, exists unique k such that $e_k(m) = c$.

Fact:

If $|\mathcal{P}| > |\mathcal{K}|$ then no scheme is perfectly secure.

The Vernam Cipher or One-time Pad

One-time Pad

Let $n \geq 1$ and $\mathcal{P}, \mathcal{C}, \mathcal{K} = \{0,1\}^n$. If $m = (m_1, \ldots, m_n) \in \mathcal{P}$ and $k = (k_1, \ldots, k_n) \in \mathcal{K}$ then

$$c = e_k(m) = (m_1 \oplus k_1, \ldots, m_n \oplus k_n).$$

Decryption: $d_k(c) = c \oplus k = m \oplus k \oplus k = m$

The One-time Pad provides perfect secrecy if used correctly:

- All keys are equally likely.
- Each key is used only once.

Two-time Pad

$$c \oplus c' = (m \oplus k) \oplus (m' \oplus k) = m \oplus m'.$$

The One-time Pad is perfectly secure but...

- The secret key must be as long as the message.
- A new key has to be generated for each communication.
- These long keys have to be exchanged in a secure way.
- Problem of generating truly random sequences for the key.

Confusion and diffusion

Diffusion: Each digit of the plaintext and each digit of the secret key should influence many digits of the ciphertext.

Confusion: The ciphertext statistics should depend on the plaintext statistics in a manner too complicated to be exploited by the cryptanalyst.

Idea: Use permutations to attain diffusion and substitutions to attain confusion.

$\rightarrow$ **Product Ciphers**

Security notions

- Perfectly secret system: the key has to be at least as long as the message.

All cryptosystems used in practice **can theoretically be broken**.

Symmetric-key approach:

Try to make the system **secure against all known attacks**.

- No attack should be faster than exhaustive search on the key.

Exhaustive search

Expected time to recover a κ-bit key: $2^{\kappa-1}$ operations.

κ (bits)	Time complexity (operations)	Security
40	2^{40}	easy to break
64	2^{64}	practical to break
80	2^{80}	not currently feasible
128	2^{128}	very strong
256	2^{256}	exceptionally strong

Table from [Knudsen, Robshaw, "*The Block Cipher Companion*", 2011.]

- The universe is less than 2^{80} microseconds old!
- The number of the protons in the universe is $\approx 2^{265}$.

Cryptanalysis of an encryption scheme

Different attack models:

- Ciphertext-only attack.
- Known-plaintext attack.
- Chosen-plaintext/ciphertext attack.
- Adaptively chosen-plaintext/ciphertext attack.

The performance of an attack is measured by its:

- time complexity.
- data complexity.
- memory complexity.

Symmetric encryption schemes

Stream ciphers

- Combine (XOR) plaintext bits with a keystream generated by a pseudo-number generator.
- Keystream should have good statistical properties.
- Advantages: Performance and low hardware complexity.

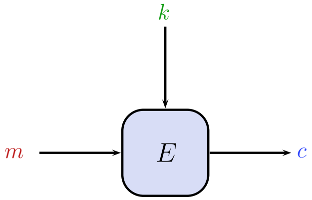
Block ciphers

- Operate on blocks of data.
- Probably the best understood symmetric primitives.
- Can be used to build hash functions, stream ciphers, MACs, authenticated encryption algorithms, PRNGs...

Block ciphers

Encrypt a block of **message** m into a block of **ciphertext** c under the action of the **key** k.

$$\begin{aligned} E : \{0,1\}^n \times \{0,1\}^\kappa &\rightarrow \{0,1\}^n \\ (m,k) &\mapsto E(m,k) = c \end{aligned}$$

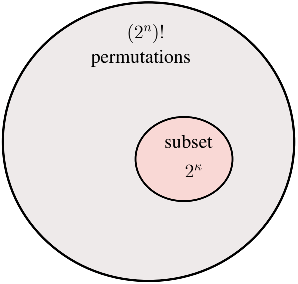


- Given k, it must be easy to compute c from m.
- Given m, c it must be hard to compute k such that $E(m,k) = c$.

Two important parameters:

- block size, n
- key size, κ

A block cipher generates a family of permutations indexed by a key k.



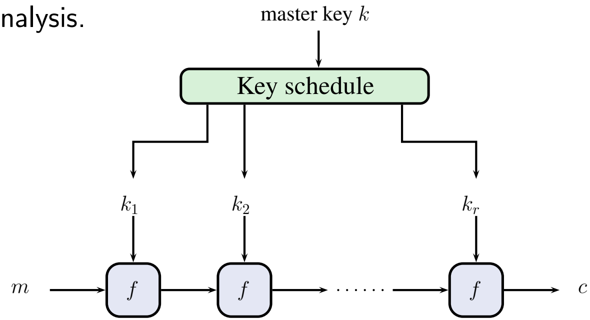
Ideal design: 2^κ permutations chosen uniformly at random from all $2^n! \approx 2^{(n-1)2^n}$ permutations.

Iterated block ciphers

Idea: Iterate a round function f several times. The function f^r is waited to be strong for large r.

Advantages:

- Compact implementation.
- Easier analysis.



Use a key schedule to extend the user-supplied (or master) key to a sequence of r subkeys.

How to build the round function?

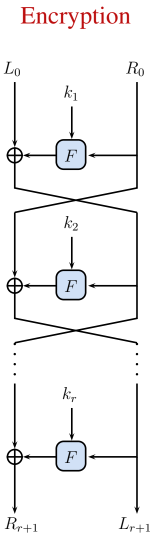
Two major approaches:

- Feistel network.
- Substitution-Permutation Network (SPN).

Feistel Network

Introduced by Horst Feistel in the early 70's.

- Split plaintext block: $m = (L_0, R_0)$
- For each round $i = 0, \ldots, r$ do:
  - $L_{i+1} = R_i$
  - $R_{i+1} = L_i \oplus F(R_i \oplus k_{i+1})$
- Ciphertext block $c = (R_{r+1}, L_{r+1})$



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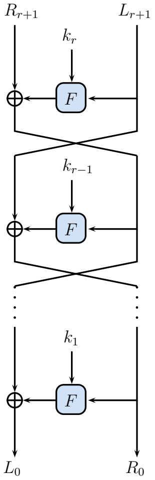
  - $L_i = R_{i+1} \oplus F(L_{i+1} \oplus k_{i+1})$
- Plaintext block $m = (L_0, R_0)$

Decryption with $K = (k_1, \ldots, k_r)$ equals encryption with $K' = (k_r, \ldots, k_1)$.

$\rightarrow$ F has not to be invertible.

Decryption



Data Encryption Standard (DES)

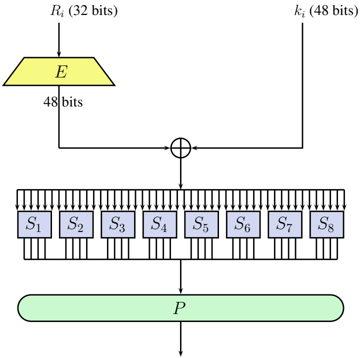
The first and probably most famous Feistel cipher.

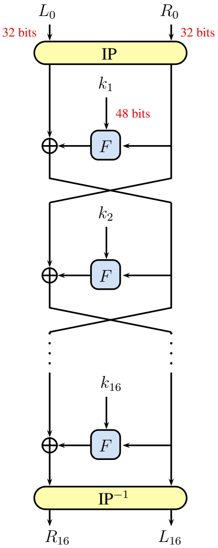
Designed by IBM and published in 1975.

- Based on an earlier internal design called *Lucifer*.
- 1977: DES is published as a FIPS standard [FIPS 46].

DES

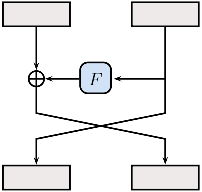
- Block size: 64 bits
- Key size: 56 bits
- 16 rounds



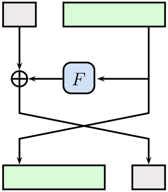


Generalized Feistel Networks

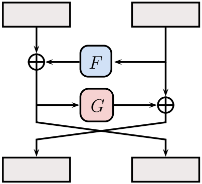
Classical Feistel



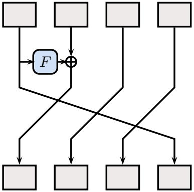
Unbalanced Feistel



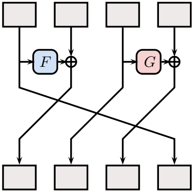
Alternating Feistel



Type-1 Feistel



Type-2 Feistel



Structrural properties of DES

The Complementation Property

$$\overline{\mathsf{DES}_k(m)} = \mathsf{DES}_{\overline{k}}(\overline{m})$$

where $\overline{x}$:= bitwise complement of x

- Limited impact to the security in the classical model.
- Halves the cost of the exhaustive key search.

Encrypt m and $\overline{m}$: $c = \mathsf{DES}_k(m)$ and $c' = \mathsf{DES}_k(\overline{m})$

For each candidate t, compute $d = \mathsf{DES}_t(m)$.

- Check if $d = c$ $\rightarrow$ t candidate for k.
- Check if $\overline{d} = c'$ ($\overline{d} = \mathsf{DES}_{\overline{t}}(\overline{m})$) $\rightarrow$ $\overline{t}$ candidate for k.

Structrural properties of DES

Weak keys

$$k \textbf{ weak}:\ \mathsf{DES}_k(\mathsf{DES}_k(m)) = m.$$

- 4 weak keys were found for DES.

Each weak key has 2^{32} **fixed points** $m : \mathsf{DES}_k(m) = m$.

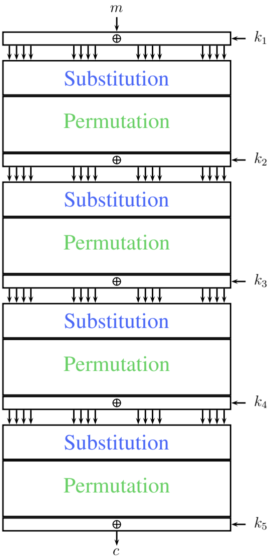
Breaking DES

- 1992 : Differential cryptanalysis (theoretical attack, 2^{47} chosen plaintexts).
- 1994 : Linear cryptanalysis (practical attack, a DES key is recovered).
- 1997: DESCHALL Project (brute-force project over the net). A message encrypted with DES is broken for the first time.
- 1999: Deep Crack and distributed.net break a DES key in less than 23 hours.
- 2004: **The standard is withdrawn.**

Key-length too short!!!

DES still survives via its Triple-DES form.

Substitution Permutation Network (SPN)



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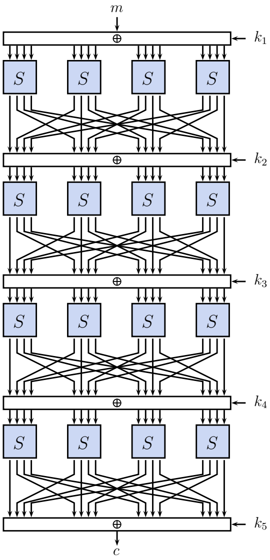


The Advanced Encryption Standard (AES) Competition [1997-2000]

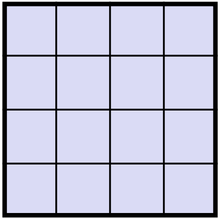
- On January 2, 1997 the NIST announced that they wished a successor to DES (to be known as AES).
- Public competition, inputs from the cryptographic community.
- Requirements: Block size of 128 bits, key size of 128, 192, 256 bits, security of 2-key triple-DES as minimum.
- 21 submissions (15 accepted for the 1st round)
- 5 finalists (Rijndael, Serpent, Twofish, RC6, MARS)
- On October 2, 2000, Rijndael becomes the AES.
- 2001: Standardization [FIPS 197]

AES

Designed by Joan Daemen and Vincent Rijmen.

Structure: Byte-oriented Substitution-Permutation Network.

- State: 128 bits, seen as a 4×4 matrix of bytes.
- 3 key-lengths: 128, 192, 256 bits
- Number of rounds: 10, 12, 14 rounds resp.



AES Representation

Each byte is viewed in two different ways:

- string of 8 bits $(b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0)$ (8^{th}-dim vector over $\mathbf{F}_2$)
- An element of the finite field with 2^8 elements $\mathbf{F}_{2^8}$

$$b_7X^7 + b_6X^6 + b_5X^5 + b_4X^4 + b_3X^3 + b_2X^2 + b_1X^1 + b_0$$

Irreducible polynomial R_P

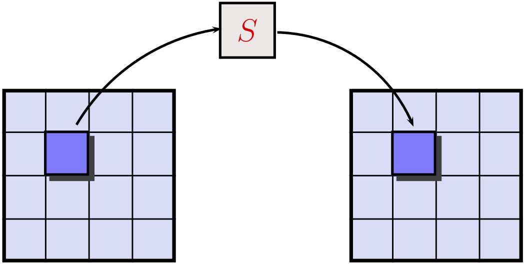
$$R_P = X^8 + X^4 + X^3 + X + 1$$

An AES round

Four byte-oriented transformations.

- `SubBytes`
- `ShiftRows`
- `MixColumns`
- `AddRoundKey`

SubBytes



The AES Sbox

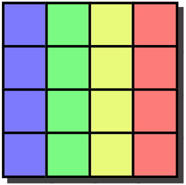
$$\begin{array}{rcl} S : \mathbf{F}_{2^8} & \rightarrow & \mathbf{F}_{2^8} \\ x & \mapsto & x^{-1} \end{array}$$

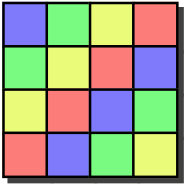
followed by an affine transformation on $\mathbf{F}_2^8$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

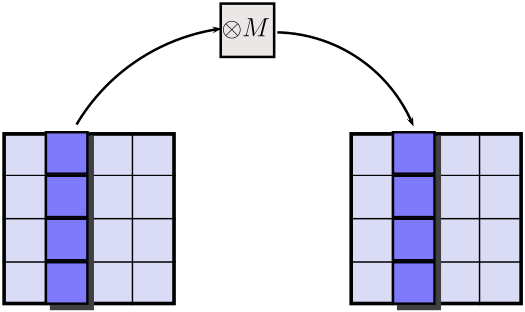
- Good resistance against differential and linear cryptanalysis.

ShiftRows



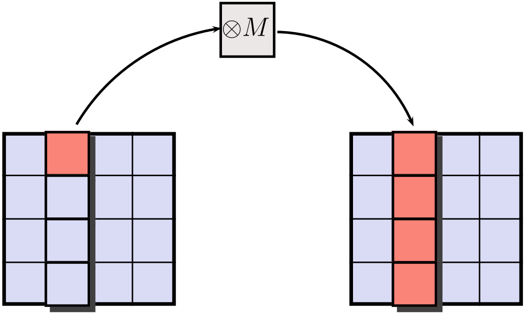


MixColumns



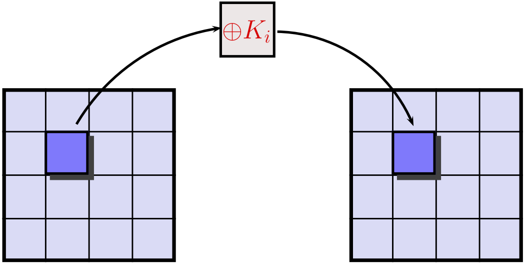
$$\begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

MixColumns



- MDS matrix.
- Branch number $= \min_{x \in \mathbf{F}_2^8}(HW(x) + HW(M(x)) = 5$.

AddRoundKey



- Lightweight non-linear key-schedule (memory, performance)

Cryptanalysis of AES

- 2000 Integral attacks
- 2002 Algebraic attacks: AES is claimed to be broken. Proved to be not realistic.
- 2009 Related-key attacks: AES-192 and AES-256 are broken under this model. Should we care?
- 2010-2013 Meet-in-the-middle attacks
- 2011 Biclique attacks: First theoretical attacks on full AES. Complexity is quite marginal (see them as accelerated exhaustive search).